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1 Electric Flux and Gauss' law

The Gauss' Law is used to find electric field when the charge is continuously distributed within an object with symmetrical geometry, such as sphere, cylinder, or plane. Gauss' law follows Coulomb's law and the Superposition Principle.

1.1 Electric Flux ϕ

Electric flux, property of an electric field that may be thought of as the number of electric lines of force (or electric field lines) that intersect a given area. Electric field lines are considered to originate on positive electric charges and to terminate on negative charges. Field lines directed into a closed surface are considered negative; those directed out of a closed surface are positive. If there is no net charge within a closed surface, every field line directed into the surface continues through the interior and is directed outward elsewhere on the surface. The negative flux just equals in magnitude the positive flux, so that the net, or total, electric flux is zero. If a net charge is contained inside a closed surface, the total flux through the surface is proportional to the enclosed charge, positive if it is positive, negative if it is negative.

The total number of electric field lines passing a given area in a unit time is defined as the electric flux

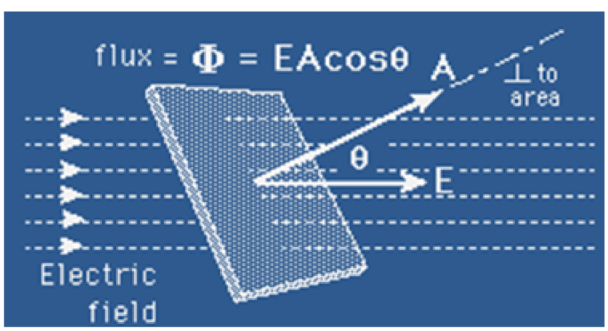
If the electric field $\vec{E}$ is uniform, the electric flux passing through a surface of vector area $\vec{A}$ is

$$\boxed{\phi = \vec{E} \cdot \vec{A}}$$
$$= EA\cos\theta$$

where θ is the angle between the electric field lines and the normal (perpendicular) to the area.

For a non-uniform electric field, the electric flux $d\phi$ through a small surface area dS is given by

$$\boxed{d\phi = \vec{E} \cdot d\vec{S}}$$
$$= EdS\cos\theta$$



Electric flux has SI units of volt metres (Vm), or, equivalently, newton metres squared per coulomb

1.2 Gauss' law

Gauss's law, also known as Gauss's flux theorem, is a law relating the distribution of electric charge to the resulting electric field.

Gauss's law is one of the four Maxwell's equations which form the basis of classical electrodynamics.Gauss's law can be used to derive Coulomb 's law, and vice versa.Gauss's law states that: The net outward normal electric flux through any closed surface is proportional to the total electric charge enclosed within that closed surface

Gauss' law states that electric flux through $\phi_{enclosed}$ any closed surface is equal to the net charge $Q_{enclosed}$ enclosed inside the surface divided by permittivity ϵ_0 of vacuum.

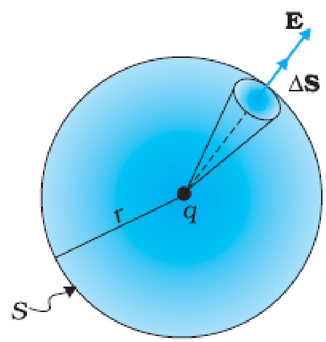
Mathematically

$$\boxed{\phi_{enclosed} = \oint \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} Q_{enclosed}}$$

Gauss's law is true for any closed surface, no matter what its shape or size. The surface that we choose for the application of Gauss's law is called the **Gaussian surface**.

1.2.1 Proof of Gauss' law

Let q be a point charge. Construct the Gaussian sphere of radius r as shown in figure.



Consider , a surface or area $\vec{ds}$. The Electric flux $d\phi$ at area element ds is

$$d\phi = \vec{E} \cdot \vec{ds} = E ds \cos\theta$$

Since filed and area vector are in the same direction, $\theta = 0$.

$$d\phi = E ds$$

So the total flux through the entire Gaussian surface of radius r is

$$\phi = \oint d\phi = \oint E ds = E \oint ds = E \cdot 4\pi r^2$$

But for a point charge q, electric filed E at a distance r is given by

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Therefore, we get

$$\phi = (\frac{1}{4\pi\epsilon_0} \frac{q}{r^2})(4\pi r^2)$$

$$\phi = \frac{q}{\epsilon_0}$$

which is the same as Gauss' law

1.2.2 Note

Inside a conductor, electrostatic field is zero. At the surface of a charged conductor, electrostatic field must be normal to the surface at every point. Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.

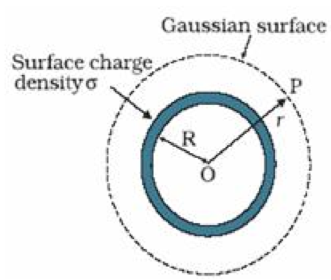
2 Electric intensity and potential due to a uniformly charged hollow sphere/thin spherical shell

Let $\sigma = \frac{q}{4\pi R^2}$ be the uniform surface charge density of a thin spherical shell of radius R. Let us find out electric field intensity at a point P outside or inside the shell.

2.1 Electric Field

2.1.1 Field outside the shell $r > R$

Consider a point P outside the spherical shell, such that $OP = r$. Consider Gaussian surface as a sphere of radius r. The electric field intensity $\vec{E}$, is same at every point of Gaussian surface, directed radially outwards and $\theta = 0$



According to Gauss' law

$$\phi_{enclosed} = \oint \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} Q_{enclosed}$$

Here

$$\oint \vec{E} \cdot d\vec{S} = Eds = E\frac{1}{4\pi r^2}$$

So

$$E\frac{1}{4\pi r^2} = \frac{q}{\epsilon_0}$$

Therefore

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

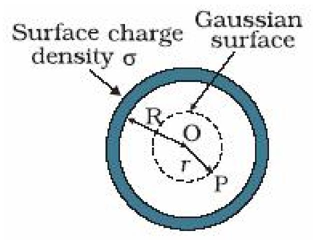
Hence it is clear that electric intensity at any point outside the spherical shell is such, as if the entire charge is concentrated at the centre of the shell.

2.1.2 Field at the surface of the shell $r = R$

At the surface of the shell, we have $r = R$. So

$$\boxed{E_{surface} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2} = \frac{\sigma}{\epsilon_0}}$$

2.1.3 Field inside the shell $r < R$



If the point P lies inside the spherical shell, then Gaussian surface is a surface of sphere of radius $r(< R)$. As there is no charge inside the spherical shell, Gaussian surface encloses no charge.

That is $Q_{enclosed} = 0$

Therefore as per Gauss' law, the field inside the spherical shell is always zero.

2.2 Potential

Potential energy of charge q at a point (in the presence of field due to any charge configuration) is the work done by the external force (equal and opposite to the electric force) in bringing the charge q from infinity to that point.

In other words, the electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point

The electric potential at a distance r from a point charge q

$$V = \frac{q}{4\pi\epsilon_0 r}$$

The relation between E and V is

$$\boxed{E = -\frac{dV}{dr}}$$

Electric field is the negative gradient of potential.

2.2.1 Potential outside the shell $r > R$

Referring the diagram as in the electric field calculation,

$$V = -\int_{\infty}^{r} E \cdot dr = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

2.2.2 Potential at the surface of the shell $r = R$

At the surface $r = R$

$$\boxed{V_{surface} = \frac{q}{4\pi\epsilon_0 R}}$$

2.2.3 Potential at the surface of the shell $r < R$

$$V = \int_{\infty}^{r} E \cdot dr = -\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr - \int_{R}^{r} 0 \cdot dr$$

$$\boxed{V_{inside} = \frac{q}{4\pi\epsilon_0 R}}$$

According to Gauss' law, there is no electric field inside a hollow conducting sphere, so moving a charge from the surface to the center takes no work. Therefore the potential difference between the surface and the center must be zero.

3 Capacitance of a parallel plate capacitor

3.1 Capacitance

A capacitor is a system of two conductors separated by an insulator. Usually, in practice, the two conductors have equal and opposite charges. The electric field in the region between the conductors is proportional to the charge Q. That is, if the charge on the capacitor is, say doubled, the electric field will also be doubled at every point. Now, potential difference V is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field. Consequently, V is also proportional to Q, and the ratio Q/V is a constant:

$$\boxed{C = \frac{Q}{V}}$$

The constant C is called the **capacitance** of the capacitor. C is independent of Q or V, as stated above. The capacitance C depends only on the geometrical configuration (shape, size, separation) of the system of two conductors. The unit is farad (F).

3.2 Parallel plate capacitor - without dielectric medium

A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance. Let intervening medium between the plates to be **vacuum**. Let A be the area of each plate and d the separation between them. The two plates have charges Q and –Q. Since d is much smaller than the linear dimension of the plates ($d^2 << A$), we can use the result on electric field by an infinite plane sheet of uniform surface charge density.

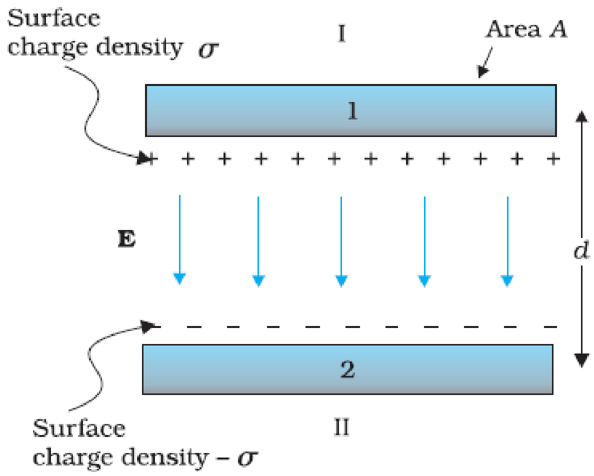


Plate 1 has surface charge density $\sigma = Q/A$ and plate 2 has a surface charge density $-\sigma$. The electric field in the Outer region I (region above the plate 1) and in the Outer region II (region below the plate 2) will be zero.

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

The direction of electric field is from the positive to the negative plate. Thus, the electric field is localised between the two plates and is uniform throughout.

Now for uniform electric field, potential difference is simply the electric field times the distance between the plates, that is,

$$V = Ed = \frac{Q}{\epsilon_0 A} d$$

The capacitance C of the parallel plate capacitor is then

$$C = \frac{Q}{V}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

3.3 Parallel plate capacitor - with dielectric medium

Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarised by the field. Now the electric field between the plates is

$$E = \frac{Q}{\epsilon_0 AK}$$

where K is a constant characteristic of the dielectric. Clearly, $K > 1$ Hence

$$V = Ed = \frac{Q}{\epsilon_0 KA} d$$

The capacitance C, with dielectric between the plates, is then

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_0 K A}{d} = K C_{vacuum}}$$

The product $\epsilon_0 K$ is called the permittivity of the medium and is denoted by $\epsilon = \epsilon_0 K$. For vacuum $K = 1$ and $\epsilon = \epsilon_0$.

ϵ_0 is called the permittivity of the vacuum. The dimensionless ratio

$=[\epsilon\frac{}{\epsilon_0}$is called the **dielectric constant of the substance**,

Thus, the dielectric constant of a substance is the factor $(K > 1)$ by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

3.4 Capacitance of a spherical conductor

An isolated charged conducting sphere has capacitance. Applications for such a capacitor may not be immediately evident, but it does illustrate that a charged sphere has stored some energy as a result of being charged.

Let a spherical conductor of radius R has a uniform charge of Q. The electric field inside a conducting sphere is zero, so the potential remains constant at the value it reaches at the surface. Then the potential on its surface is

$$V = \frac{Q}{4\pi\epsilon_0 R}$$

Therefore

$$\boxed{C = \frac{Q}{V} = 4\pi\epsilon_0 R}$$

4 Biot & Savart's law

All magnetic fields that we know are due to currents (or moving charges) and due to intrinsic magnetic moments of particles. The relation between current and the magnetic field it produces is given by the Biot-Savart's law.

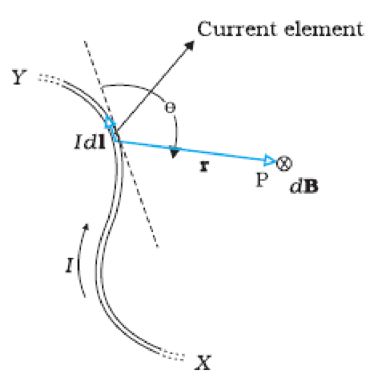


Figure shows a finite conductor XY carrying current I. Consider an infinitesimal element dl of the conductor. The magnetic field dB due to this element is to be determined at a point P which is at a distance r from it. Let ϕ be the angle between dl and the displacement vector r. According to Biot-Savart's law, the magnitude of the magnetic field dB is proportional to the current I, the element length

$|dl|$, and inversely proportional to the square of the distance r. Its direction is perpendicular to the plane containing dl and r .

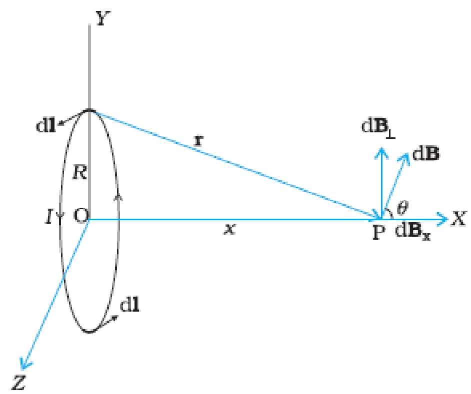
$$\boxed{dB \propto \frac{I dl \sin\phi}{r^2}}$$

$$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I dl \sin\phi}{r^2}}$$

We call μ_0 the permeability of free space (or vacuum).

4.1 Field along the axis of a circular coil carrying current

Figure depicts a circular loop carrying a steady current I. The loop is placed in the $y - z$ plane with its centre at the origin O and has a radius R. The x-axis is the axis of the loop. We wish to calculate the magnetic field at the point P on this axis. Let x be the distance of P from the centre O of the loop.



Consider a conducting element dl of the loop. The magnitude dB of the magnetic field due to dl is given by the Biot-Savart law

$$dB = \frac{\mu_0}{4\pi}\frac{I dl \sin\phi}{r^2}$$

Now $r^2 = x^2 + R^2$. Further, any element of the loop will be perpendicular to the displacement vector from the element to the axial point, so $\phi = 90^0$.

The direction of dB is shown in Fig. It is perpendicular to the plane formed by dl and r. It has an x-component dB_x and a component perpendicular to x-axis, dB_{perp}. When the components perpendicular to the x-axis are summed over, they cancel out and we obtain a null result. Thus, only the x-component survives. The net contribution along x-direction can be obtained by integrating $dB_x = dB\cos\theta$ over the loop.

$$\cos\theta = \frac{R}{(x^2 + R^2)^{1/2}}$$

Therefore, the total field at P due to the entire current loop will be obtained by integrating dB_x

$$B = \oint dB_x = \oint \frac{\mu_0}{4\pi} I dl \frac{R}{(x^2 + R^2)^{1/2}}$$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2\pi R^2}{(x^2 + R^2)^{1/2}}}$$

As a special case of the above result, we may obtain the field at the centre of the loop. Here $x = 0$, and we obtain,

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

4.2 Force on current carrying conductor placed in a magnetic field

Consider a rod of a uniform cross-sectional area A and length l. Let the number density of mobile charge carriers (electrons) in it be n. Then the total number of mobile charge carriers in it is nAl. For a steady current I in this conducting rod, we may assume that each mobile carrier has an average drift velocity v_d. In the presence of an external magnetic field B, the force on these carriers is:

$$F = (nAl)q\vec{v_d} \times \vec{B}$$

where q is the value of the charge on a carrier. Now nqv_d is the current density j and $|(nqv_d)|A$ is the current I.

Thus,

$$F = [(nqev_d)Al] \times \vec{B} = [jAl] \times \vec{B}$$

$$\boxed{F = I\vec{l} \times \vec{B}}$$

where $\vec{l}$ is a vector of magnitude l, the length of the rod, and with a direction identical to the current I. The current I is not a vector. The equation holds for a straight rod. In this equation, B is the external magnetic field. It is not the field produced by the current-carrying rod.

4.3 Theory of moving coil galvanometer

The moving coil galvanometer (MCG) is an electromagnetic device that can measure small values of current.

The galvanometer consists of a coil, with many turns, free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it. This torque is given by

$$\tau = NIAB$$

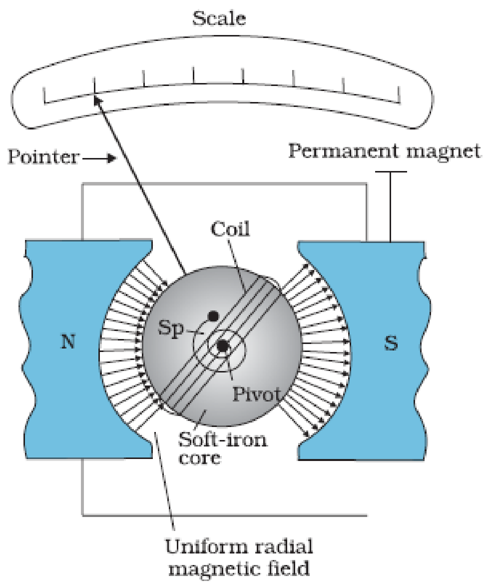
where the symbols have their usual meaning. Since the field is radial by design, we have taken $\sin\theta = 1$ in the above expression for the torque. The magnetic torque tends to rotate the coil. A spring Sp provides a counter torque $k\phi$ that balances the magnetic torque $NIAB$; resulting in a steady angular deflection ϕ. In equilibrium

$$k\phi = NIAB$$

where k is the torsional constant of the spring; i.e. the restoring torque per unit twist. The deflection ϕ is indicated on the scale by a pointer attached to the spring. We have

$$\boxed{\phi = \frac{NAB}{k} I}$$

The quantity in brackets is a constant for a given galvanometer.



The galvanometer can be used in a number of ways. It can be used as a detector to check if a current is flowing in the circuit. In this usage the neutral position of the pointer (when no current is flowing through the galvanometer) is in the middle of the scale and not at the left end. Depending on the direction of the current, the pointer deflection is either to the right or the left.

The galvanometer cannot as such be used as an ammeter to measure the value of the current in a given circuit. This is for two reasons: (i) Galvanometer is a very sensitive device, it gives a full-scale deflection for a current of the order of μA. (ii) For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of the current in the circuit. To overcome these difficulties, one attaches a small resistance r_s, called shunt resistance, in parallel with the galvanometer coil; so that most of the current passes through the shunt.

We define the current sensitivity of the galvanometer as the deflection per unit current. The current sensitivity is

$$\frac{\phi}{I} = \frac{NAB}{k}$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N. We choose galvanometers having sensitivities of value, required by our experiment. The galvanometer can also be used as a voltmeter to measure the voltage across a given section of the circuit. For this it must be connected in parallel with that section of the circuit. Further, it must draw a very small current, otherwise the voltage measurement will disturb the original set up by an amount which is very large. Usually we like to keep the disturbance due to the measuring device below one per cent. To ensure this, a large resistance R is connected in series with the galvanometer.

5 Questions

1. What is Electric Flux ?
2. State and Explain Gauss' law in electrostatics
3. What is the use of Gauss' law in electrostatics ?
4. What is a Gaussian surface ?
5. What is electrostatic potential ?
6. What is electric field ?
7. Derive expressions for Electric intensity and potential due to a uniformly charged hollow sphere
8. What is meant by capacitance ? Give its unit
9. Give the relation connecting capacitance and potential
10. Derive the expressions for capacitance of Parallel plate capacitor - with and without dielectric medium
11. State and explain Biot & Savart's law
12. Derive an expression for Field along the axis of a circular coil carrying current
13. Derive an expression fo Force on current carrying conductor placed in a magnetic field
14. What is the working principle of a moving coil galvanometer
15. Define the terms current sensitivity and voltage sensitivity of a moving coil galvanometer